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# Section 1

4. Let Q(n) be the predicate “n^2 ≤ 30.”

a. Write Q(2), Q(−2), Q(7), and Q(−7), and indicate which of these statements are true and which are false.

Q(2) (2)^2≤30 => 4 ≤30 true

Q(-2) (-2)^2≤30 => 4 ≤30 true

Q(7) (7)^2≤30 => 49 ≤30 false

Q(-7) (-7)^2≤30 => 49 ≤30 false

b. Find the truth set of Q(n) if the domain of n is Z, the set of all integers.

If the domain of Q(n) is the set of all integers, then its

truth set is {−5,−4,−3,−2,−1, 0, 1, 2, 3, 4, 5}.

c. If the domain is the set Z+ of all positive integers, what is the truth set of Q(n)?

If the domain of Q(n) is the set of all positive integers, then its

truth set is {0, 1, 2, 3, 4, 5}.

8. Let B(x) be “−10 < x < 10.” Find the truth set of B(x) for each of the following domains.

a. Z

{−9,−8,−7,−6,−5,−4,−3,−2,−1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

b. Z+

{0, 1, 2, 3, 4, 5,6, 7, 8, 9}

c. The set of all even integers

{−8,−6,−4,−2, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Find counterexamples to show that the statements in 12 are

false.

12. ∀ real numbers x and y,

√(x + y) =√x +√y

Let x=1, y=2

√3=1+√2

(This is one counter example)

16. Rewrite each of the following statements in the form “∀ x.”

a. All dinosaurs are extinct.

∀ dinosaurs x, x is a extinct.

b. Every real number is positive, negative, or zero.

∀ real number x, x is either positive, negative or zero.

c. No irrational numbers are integers.

∀ irrational x, x is not an integer.

d. No logicians are lazy.

∀ logicians x, x is not lazy.

e. The number 2,147,581,953 is not equal to the square of

any integer.

∀ integers x, x2 does not equal 2, 147, 581, 953.

f. The number −1 is not equal to the square of any real

∀ real number x, x2 does not equal -1.

20. Rewrite the following statement informally in at least two

different ways without using variables or the symbol ∀ or

the words “for all.”

∀ real numbers x, if x is positive, then the square root of x is positive.

a. the square root of all positive real numbers is positive

b. if a real number is positive then its square root is positive

24. Rewrite the following statements in the two forms

“∃ x such that ” and “∃x such that and .”

a. Some hatters are mad.

∃ a hatter x such that x is mad.

∃x such that x is a hatter and x is mad.

b. Some questions are easy.

∃ a question x such that x is easy.

∃x such that x is a question and x is easy.

28. Let the domain of x be the set Z of integers, and let Odd(x)be “x is odd,” Prime(x) be “x is prime,” and Square(x) be “x is a perfect square.” (An integer n is said to be a perfect square if, and only if, it equals the square of some integer.

For example, 25 is a perfect square because 25 = 52.)

a. ∃x such that Prime(x)∧ ∼Odd(x).

There exist a integer that is prime and is not odd

b. ∀x, Prime(x)→∼Square(x).

If an integer is prime, then itis not a perfect square.

c. ∃x such that Odd(x) ∧ Square(x).

There exist an integer that odd and its square

32. Let R be the domain of the predicate variable x. Which of

the following are true and which are false? Give counter

examples for the statements that are false.

a. x > 2 ⇒ x >1 True: Any real number that is greater than 2 is greater than 1.

b. x > 2 ⇒ x2 > 4 True: Any real number greater than 2 have his square greater than 4

c. x2 > 4 ⇒ x > 2 False: (−3)2 > 4 but −3 ≯ 2.

d. x2 > 4 ⇔ |x| > 2 True

# Section 2

6. Write a negation for each of the following statements.

a. Sets A and B do not have any points in common.

Sets A and B have at least one point in common.

b. Towns P and Q are not connected by any road on the map.

Towns P and Q have many connected road on the map.

determine whether the proposed negation is correct. If it is not, write a correct negation

12. Statement: The product of any irrational number

and any rational number is irrational.

Proposed negation: The product of any irrational number

and any rational number is rational.

No

Negation: There exist a product of a rational number and any rational number that is not rational

write a negation for each statement.

18. ∀x ∈ R, if x(x + 1) > 0 then x > 0 or x < −1.

∃ a real number x such that x(x + 1) > 0 and both x ≤ 0

and x ≥ −1.

24. Rewrite the statements in each pair in if-then form and indicate the logical relationship between them.

a. All the children in Tom’s family are female.

All the females in Tom’s family are children.

If a person is a child in Tom’s family, then the person is female.

If a person is a female in Tom’s family, then the person

is a child.

b. All the integers that are greater than 5 and end in 1, 3, 7, or 9 are prime.

All the integers that are greater than 5 and are prime end in 1, 3, 7, or 9.

If an integer greater than 5 and is prime then it ends with 1, 3, 7, or 9.

If an integer greater than 5 ends in 1, 3, 7, or 9, then the number is prime.

In 26–33, for each statement in the referenced exercise write the converse, inverse, and contrapositive. Indicate as best as you can which among the statement, its converse, its inverse, and its contrapositive are true and which are false. Give a counterexample for each that is false.

30. Exercise 20

Statement: ∀ integers a, b, and c, if a − b is even and b – c is even, then a − c is even.

Contrapositive: ∀ integers a, b, and c, if a − c is not even, then a − b is not even or b − c is not even.

Converse: ∀ integers a, b and c, if a − c is even then a – b is even and b − c is even.

Inverse: ∀ integers a, b, and c, if a − b is not even or b – c is not even, then a − c is not even. The statement is true, but its converse and inverse are false.

Counterexample, let a = 3, b = 2, and c = 1. Then a − c = 2, which is even, but a − b = 1 and b − c = 1, so it is not the case that both a − b and b − c are even.

36. If P(x) is a predicate and the domain of x is the set of all real numbers, let R be “∀x ∈ Z, P(x),” let S be “∀x ∈ Q, P(x),” and let T be “∀x ∈ R, P(x).”

a. Find a definition for P(x) (but do not use “x ∈ Z”) so that R is true and both S and T are false.

Let P(x) be 2x does not equal 1.

The statement ∀x ∈ Z, 2x does not equal 1 is true, but the statements ∀x ∈ Q, 2x does not equal 1 and ∀x ∈ R, 2x does not equal 1 are both false.

b. Find a definition for P(x) (but do not use “x ∈ Q”) so that both R and S are true and T is false.

Rewrite each statement of 42 in if-then form.

42. Passing a comprehensive exam is a necessary condition for obtaining a master’s degree.

If you pass a comprehensive exam, then you can obtain a master’s degree

48. A frequent-flyer club brochure states, “You may select among carriers only if they offer the same lowest fare.” Assuming that “only if” has its formal, logical meaning, does this statement guarantee that if two carriers offer the same lowest fare, the customer will be free to choose between them? Explain.

No. Interpreted formally, the statement says, “If carriers do not offer the same lowest fare, then you may not select among them,” or, equivalently, “If you may select among carriers, then they offer the same lowest fare.”

# Section 3

10. This exercise refers to Example 3.3.3. Determine whether each of the following statements is true or false.

a. ∀ students S, ∃ a dessert D such that S chose D.

True. Every student chose at least one dessert

b. ∀ students S, ∃ a salad T such that S chose T.

False: This statement says that every student chose a salad, Yuen did not.

c. ∃ a dessert D such that ∀ students S, S chose D.

True: Every student chose pie

d. ∃ a beverage B such that ∀ students D, D chose B.

False: There is not a beverage that every student choose

e. ∃ an item I such that ∀ students S, S did not choose I.

False: every item was chose at least once

f. ∃ a station Z such that ∀ students S, ∃ an item I such that S chose I from Z.

True: there are 3 stations where every student chose at less one item

20. Recall that reversing the order of the quantifiers in a statement with two different quantifiers may change the truth value of the statement—but it does not necessarily do so. All the statements in the pairs on the next page refer to the Tarski world of Figure 3.3.1. In each pair, the order of the quantifiers is reversed but everything else is the same. For each pair, determine whether the statements have the same or opposite truth values. Justify your answers.

a. (1) For all squares y there is a triangle x such that x and y have different color.

(2) There is a triangle x such that for all squares y, x and y have different colors.

Statement (1) says that no matter what square anyone might give you, you can find a triangle of a different color. This is true because the only squares are e, g, h, and j, and given squares g and h, which are gray, you could take triangle d, which is black; given square e, which is black, you could take either triangle f or i , which are gray; and given square j , which is blue, you could take either triangle f or h, which are gray, or triangle d, which is black.

b. (1) For all circles y there is a square x such that x and y have the same color.

(2) There is a square x such that for all circles y, x and y have the same color.

Statement (1) says that no matter what circle anyone might give you, you can find a square with the same color. This is true because there are only three circles and circle c and a have square j which have the same color and circle b have square g

For each of the statements in 29 and 30, (a) write a new statement by interchanging the symbols ∀ and ∃, and (b) state which is true: the given statement, the version with interchanged quantifiers, neither, or both.

30. ∃x ∈ R such that ∀y ∈ R− (the set of negative real numbers), x > y.

a) ∀x ∈ R such that ∃y ∈ R− (the set of negative real numbers), x > y.

b) Both statements are true.

40. In informal speech most sentences of the form “There is every ” are intended to be understood as

meaning “∀ ∃ ,” even though the existential quantifier there is comes before the universal quantifier

every. Note that this interpretation applies to the following well-known sentences. Rewrite them using quantifiers and variables.

a. There is a sucker born every minute.

∀ minutes m, ∃ a sucker s such that s was born in minute m.

b. There is a time for every purpose under heaven.

∀ purpose p, ∃ a time t such that t for p.

refer to the Tarski world given in Figure 3.1.1. The domains of all variables consist of all the objects in the Tarski world. For each statement,

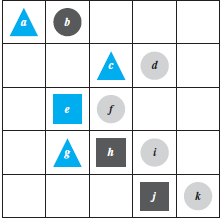
50. For every object x, there is an object y such that if x does not equal y then x and y have different colors.

(a) indicate whether the statement is true or false and justify your answer

(b) write the given statement using the formal logical notation illustrated in Example 3.3.10, and

(c) write the negation of the given statement using the formal logical notation of Example 3.3.10.

find the answers Prolog would give if the following questions were added to the program given in Example 3.3.11.

60. 

a. ?isabove(b1,w1) Yes

b. ?color(X, white) X = w1, X = w2

c. ?isabove(X, b3) X = b2, X = w2

# Section 4

Use universal instantiation or universal modus ponens to fill in valid conclusions for the arguments in 2–

4. 4. ∀ real numbers r , a, and b, if r is positive, then (ra)b = r ab.

r = 3, a = 1/2, and b = 6 are particular real numbers such that r is positive.

∴ (31/2)6 = 36/2 => 33= 33

Some of the arguments in 7-18 are valid by universal modus ponens or universal modus tollens; others are invalid and exhibit the converse or the inverse error. State which are valid and which are invalid. Justify your answers.

8. All freshmen must take writing.

Caroline is a freshman.

∴ Caroline must take writing.

Valid by universal modus ponens (or universal instantiation)

12. All honest people pay their taxes.

Darth is not honest.

∴ Darth does not pay his taxes.

Invalid by inverse error

**16.** If a number is even, then twice that number is even.

The number 2*n* is even, for a particular number *n*.

∴ The particular number *n* is even.

Invalid; converse error

20. a. Use a diagram to show that the following argument can have true premises and a false conclusion.

All dogs are carnivorous.

Aaron is not a dog. -Aron

∴ Aaron is not carnivorous. Carnivorous

Dog

b. What can you conclude about the validity or invalidity of the following argument form? Explain how the result

from part (a) leads to this conclusion.

∀*x,* if *P(x)* then *Q(x).*

∼*P(a)* for a particular *a.*

∴ ∼*Q(a).*

It is invalid because of Inverse Error

Indicate whether the arguments in 24 are valid or invalid. Support your answers by drawing diagrams.

24. No vegetarians eat meat.

All vegans are vegetarian.

∴ No vegans eat meat. Don’t Eat meat

Valid

Vegetarian

Vegans

In exercises 28–32, reorder the premises in each of the arguments to show that the conclusion follows as a valid consequence from the premises. It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives. Exercises 28–30 refer to the kinds of Tarski worlds discussed in Example 3.1.13 and 3.3.1. Exercises 31 and 32 are adapted from *Symbolic Logic* by Lewis Carroll.∗

**28.**

1. Every object that is to the right of all the blue objects is

above all the triangles.

2. If an object is a circle, then it is to the right of all the

blue objects.

3. If an object is not a circle, then it is not gray.

∴ All the gray objects are above all the triangles.

(3) *Contrapositive form:* If an object is gray, then it is a

circle.

(2) If an object is a circle, then it is to the right of all the

blue objects.

(1) If an object is to right of all the blue objects, then it is

above all the triangles.

∴ If an object is gray, then it is above all the triangles.

32.

1. When I work a logic example without grumbling, you may be sure it is one I understand.

2. The arguments in these examples are not arranged in regular order like the ones I am used to.

3. No easy examples make my head ache.

4. I can’t understand examples if the arguments are not arranged in regular order like the ones I am used to.

5. I never grumble at an example unless it gives me a headache.

∴ These examples are not easy.